

3. É. A. Zakharova et al., "Calculation of the true vapor content in heated channels," Teploenergetika, No. 6 (1970).
4. R. W. Bowring, "Physical model based on bubble detachment and calculation of steam voidage in the subcooled region of a heated channel," HPR-10 (December, 1962).
5. B. R. Bergel'son and A. S. Gerasimov, Preprint No. 17, Inst. Teor. Eksp. Fiz. (1977).
6. V. A. Knyazev, "Hydraulic resistance in channels with surface boiling," At. Energ., 38, No. 1 (1975).
7. L. Ya. Kramerov and Ya. V. Shevelev, Engineering Problems of Reactor Construction [in Russian], Gosatomizdat, Moscow (1966).
8. R. Forgan and R. Whittle, "Pressure-drop characteristics for the flow of subcooled water at atmospheric pressure in narrow heated channels. I and II," AERE-M1739 (1966).
9. I. K. Ferrell, "A study of calculation on boiling inside a channel," North Carolina State Univ., Raleigh (September, 1946).
10. L. Cimorelli and A. Premoli, CNEN Energ. Nucl., 13, No. 1 (1966).
11. G. B. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill, New York (1969).

TURBULENT-FLOW RESISTANCE FOR SOLUTIONS OF POLYMERS
AND MICELLE-FORMING SURFACTANTS

I. L. Povkh, V. G. Pogrebnyak, and A. I. Toryanik

UDC 532.517.4

Calculation method and results are presented for the hydrodynamic-resistance coefficients of solutions of polymers and surfactants.

There are various methods of reducing resistance such as boundary-layer suction and injection, vibration of the surface with an appropriate frequency and amplitude, etc., but in recent years considerable attention has been given to the use of polymers and micelle-forming surfactants in turbulent flows. Many different points of view have been expressed [1-10] on the mechanism.

On our view, the reduced hydrodynamic resistance in solutions of polymers and surfactants is due to the anisotropy in the viscosity arising in the viscous sublayer and the transitional layer on account of deformation and orientation of the macromolecules and micelles along the flow, i.e., the differences in resistance to the displacement and growth of turbulent eddies in the various directions. The elevated shear viscosity in the perpendicular direction results in additional resistance to pulsation and therefore reduces the general level of mixing. Therefore, the pulsation frequency in polymer and surfactant solutions tends to be lower than that in the solvent at the same Reynolds number, i.e., the generation of turbulence is reduced and the turbulent dissipation in the flow is lower.

Here we present calculations on this basis for the reduction in turbulent friction for polymers and surfactants, and a comparison is made with experiment.

The following formula [11] is used to derive the resistance law for polymers and surfactants:

$$\Delta P_0 = \lambda \frac{l}{2R} \frac{\rho \bar{U}^2}{2} \quad (1)$$

Formula (1) relates the pressure difference ΔP_0 across a cylindrical tube of length l and radius R to the resistance coefficient λ , the geometrical dimensions of the part of the tube, the density ρ , and the mean flow velocity \bar{U} . We substitute in (1) the Reynolds number $Re = 2R\bar{U}/\nu$ (ν is the kinematic viscosity) to get that the pressure difference in laminar flow is given by

$$\Delta P_{0l} = A\rho Re^2 \lambda_l \nu^2, \quad (2)$$

Donets University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 5, pp. 793-797, November, 1979. Original article submitted May 17, 1979.

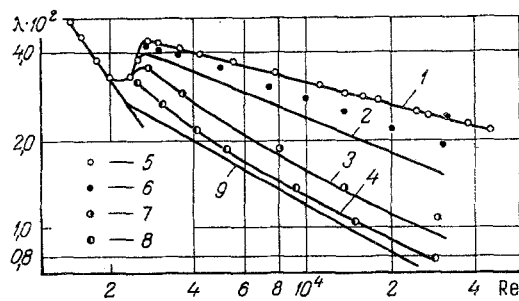


Fig. 1. Resistance coefficient for aqueous polyethylene oxide solutions of molecular mass $3 \cdot 10^6$ and various concentrations in relation to Reynolds number at 20°C : 1-4) theoretical curves; 5-8) observed points; 1,5) H_2O ; 2,6) 0.0001%; 3,7) 0.002%; 4,8) 0.003%; 9) Virk asymptote.

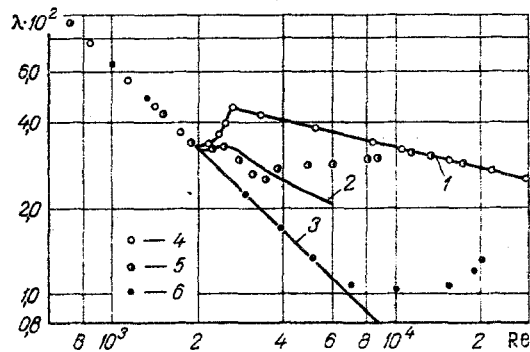


Fig. 2. Resistance coefficient for water-salt (7.5% KCl, pH = 11) solutions of potassium oleate in relation to Reynolds number at 20°C : 1-3) theoretical curves; 4-6) observed points; 1 and 4) H_2O ; 2 and 5) 0.05%; 3) and 6) 0.2%.

while for turbulent flow it is given by

$$\Delta P_{ot} = A \rho Re^2 \lambda_t v^2, \quad (3)$$

where $A \equiv l/8R^3$.

The difference ΔP_T between these values for turbulent and laminar modes is governed by the rate of mixing due to the pulsation; an increase in shear viscosity perpendicular to the flow such as arises in oriented media reduces ΔP_T for solutions of polymers and surfactants and governs the reduction in the hydrodynamic resistance.

To incorporate these features of polymers and surfactants we consider two values of the viscosity, ν_y and ν_x [11]: Since there is a higher velocity gradient near the wall, the long macromolecules and micelles are oriented along the flow; this increases the shear viscosity perpendicular to the flow (ν_y) while decreasing it in the longitudinal direction ν_x . Then (2) takes the following form for polymers and surfactants:

$$\Delta P_{otp} = A \rho Re^2 \lambda_p \nu_x^2. \quad (4)$$

Since an increase in ν_y should reduce ΔP_{Tp} , we get

$$\Delta P_{Tp} = f(\nu) / \nu_y^2. \quad (5)$$

The function $f(\nu)$ is derived from the condition that if $\nu_y = \nu_x$, then $\Delta P_{Tp} = \Delta P_{Tw}$, so $f(\nu) = \Delta P_{Tw} \nu_x^2$; we substitute this into (5) to get

$$\Delta P_{Tp} = \Delta P_{Tw} \frac{v_x^2}{v_y^2} = \frac{\Delta P_{Tw}}{K_a^2}, \quad (6)$$

where K_a is the anisotropy coefficient for the viscosity. Subscript w relates to the solvent (water), while subscript p relates to the polymer solution. Then if we know the anisotropy coefficient and the difference between the pressures for the solvent, we can determine the pressure difference for polymers and surfactants.

The relative change in λ reflects the fall in turbulent resistance and can be put as

$$\frac{\lambda_{Tw} - \lambda_{Tp}}{\lambda_{Tw}} \equiv \frac{\Delta \lambda}{\lambda} = \left[1 - \frac{\Delta P_{oTp}}{\Delta P_{oTw}} \left(\frac{t_p}{t_w} \right)^2 \right] \cdot 100 \%, \quad (7)$$

where t is the time taken for a given volume of liquid to flow through the tube.

From (2)-(7) we get the resistance reduction in relation to the anisotropy coefficient for three cases:

$$1) \bar{U}_p = \bar{U}_w$$

$$\frac{\Delta \lambda}{\lambda} = 1 - \frac{\lambda_{Tw} + (\lambda_{Tw} - \lambda_{Tw}) / \bar{K}_a^2}{\lambda_{Tw}}; \quad (8)$$

$$2) Re_p = Re_w$$

$$\frac{\Delta \lambda}{\lambda} = 1 - \frac{\lambda_{Tw} + (\lambda_{Tw} - \lambda_{Tw}) / \bar{K}_a^2 \left(\frac{v_x}{v_w} \right)^2}{\lambda_{Tw}}; \quad (9)$$

$$3) \Delta P_{oTp} = \Delta P_{oTw}$$

$$\frac{\Delta \lambda}{\lambda} = 1 - \left(\frac{\lambda_{Tw} + (\lambda_{Tw} - \lambda_{Tw}) / \bar{K}_a^2}{\lambda_{Tw}} \right)^2. \quad (10)$$

The resistance coefficients λ_{Tw} and λ_{Tw} are calculated from standard equations [11]:

$$\lambda_{Tw} = 0,3164 / Re^{0,25}, \quad (11)$$

$$\lambda_{Tw} = 64 / Re. \quad (12)$$

We determine $\Delta \lambda / \lambda$ from one of the equations (8)-(10) and use (11) to get the resistance coefficients for the solutions, i.e., we get the resistance law for the polymer and surfactant solutions from the resistance law for the solvent.

The viscosity anisotropy in the turbulent flow occurs in the viscous sublayer and in transitional layer, so in (8)-(10) we have introduced the effective anisotropy coefficient \bar{K}_a , which is averaged over the thickness of the boundary layer.

In the case of a dilute solution of linear flexible-chain macromolecules, we can estimate the true anisotropy coefficient from the theory of anisotropic hydrodynamic interaction [12-15], which gives a good description of the effects of shear velocity on the characteristic viscosity for aqueous solutions of polyethylene oxide [10]. On that theory, the limiting value of δ (as $g \rightarrow \infty$) is $\delta_\infty = f_x / f_y - 1 = -1/2$. Since the coefficient of friction f is proportional to the viscosity, we have $\delta = v_x / v_y - 1$, and, therefore, since $K_a = v_y / v_x$ the anisotropy coefficient expressed as a function of δ in the form $K_a = 1 / (1 + \delta)$. Thus $K_a = 2$ for the concentration at which the reduction is optimal. For flows in tubes of small diameter we put $K_a \approx \bar{K}_a$ and use one of the equations (8)-(10) to get the maximum reduction in the hydrodynamic resistance in the flow of a dilute flexible-chain polymer solution as 75%, which agrees with the experimental evidence [16].

We can represent K_a as v_0 / v_x for concentrated surfactant solutions, where v_0 is the viscosity at zero shear stress and v_x is the value corresponding to the shear stress for the given Reynolds number (K_a can be measured directly by the method of [17]).

Figures 1 and 2 show resistance coefficients for aqueous solutions of polyethylene oxide of molecular mass $3 \cdot 10^6$ and for water-salt (7.5% KCl, pH = 11) solutions of potassium oleate, respectively, for flow in a tube of diameter $2.68 \cdot 10^{-3}$ m (the anisotropy coefficient for the polyethylene oxide solutions was calculated from the above theory). There is good agreement between the calculated and observed resistance coefficients in the region of optimum resistance reduction.

NOTATION

ΔP , pressure drop, N/m²; l , length, m; R , radius, m; λ , drag coefficient; ρ , density, kg/m³; \bar{U} , mean flow speed, m/sec; ν , kinematic viscosity, m²/sec; K_a , anisotropy coefficient; Re , Reynolds number; f , coefficient of friction; g , velocity gradient, sec⁻¹; t , time, sec.

LITERATURE CITED

1. J. Hoyt, *Trans. Am. Soc. Mech. Eng.*, 94, No. 2, 1 (1972).
2. Y. L. Lumley, *Ann. Rev. Fluid Mech.*, No. 1, 367 (1969).
3. G. I. Barenblatt and V. M. Kalashnikov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 68 (1968).
4. N. G. Basetskaya and V. A. Ioselevich, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2, 110 (1970).
5. G. F. Kobets, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 1, 107 (1969).
6. I. L. Povkh and A. B. Stupin, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 1, 63 (1972).
7. V. A. Nikulin, in: *Physical Hydrodynamics [in Russian]*, Vishcha Shkola, Kiev-Donetsk (1977), p. 34.
8. S. Ya. Frenkel', in: *Physics Today and Tomorrow [in Russian]*, Nauka, Leningrad (1973), p. 176.
9. S. Ya. Frenkel, *Pure Appl. Chem.*, 38, 117 (1974).
10. V. G. Pogrebnyak, *Author's Abstract of Candidate's Dissertation*, Khark'kov (1979).
11. I. L. Povkh, *Engineering Hydromechanics [in Russian]*, Mashinostroenie, Leningrad (1976).
12. A. Peterlin and M. Čopić, *J. Appl. Phys.*, 27, 434 (1959).
13. M. Čopić, *J. Chim. Phys.*, 53, 440 (1956).
14. M. Čopić, *J. Chim. Phys.*, 53, 348 (1957).
15. Y. Ikeda, *J. Phys. Soc. Jpn.*, 12, 378 (1957).
16. P. S. Virk, H. S. Mickicy, and K. A. Smith, *Am. Soc. Mech. Eng. J. Appl. Mech.*, 37, 488 (1970).
17. A. Ya. Malkin et al., *Kolloidn. Zh.*, No. 1, 200 (1979).